CONTENTS

Analytical approach to the calculation procedure of N.A. on paragraph 4.2.3.2.(8) EN 1998-1

1. General

The Greek national annex on EN 1998-1 restores the definition of mass eccentricity[1] in accordance with the requirements that existed in Greek Earthquake Standard (EAK2000) paragraph 3.3.3. and Annex ΣΤ. In the following it is attempted a logical sequence for computing the figures, mainly as a memorandum on how exactly things come out[2].

In the following the term "a dynamically independent unit" means a building within the notice of the paragraph 4.2.3.1. (1) P.

2. Geometry of storeys in plan

For each storey are calculated, in the reference coordinate system:

- The values of moment of inertia I_{X0}, I_{Y0} and I_{XY0} of the shape in plan of the diaphragm. Aim of this calculation is the determination of polar moment of inertia at the center of mass ($I_{Pi}=I_{Xi}+I_{Yi}$), using the Steiner transformation formula from the origin of the reference system.
- Determination of storey total mass M_i and centre of mass (X_M,Y_M).

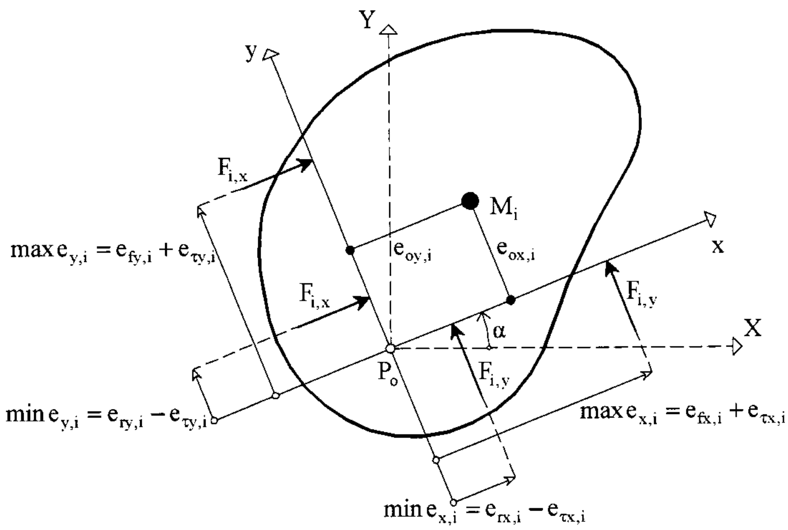
3. Determination of "elastic" axis of the building

It is defined a real or virtual axis of the building as follows:

- Determine the nearest building diaphragm (reference diaphragm) to the level z_o=0.8H, where H[3] is the building height.
- Load diaphragms with, all having the same sign,(torsional) moments $M_{zi}=cF_i$ where, F_i[4] is the static storey seismic force and c[5] an arbitrary number.
- Calculate the angle of twist of every diaphragm θ_{zi} and find the pole of the reference diaphragm P_o (X_{PO},Y_{PO}). In every other diaphragm consider the points where P_o is projected. This shall be considered as center of inertia.

[1] The introduced, by N.A., method practically leads to spatial models for building calculations because requires the development of a such for the calculation of these parameters.
[2] Footer notes(in blue) express personal positions of the author of this text.
[3] The height of the building is not clearly determined (e.g. if consider the end of staircase shed at the terrace this may change the reference diaphragm with almost no change in model). Also it is not explained what to select, in case that level z_0 is found exactly between two diaphragms.
[4] The static force F_i is defined on paragraph 4.3.3.2.3.(2)P and (3) EN 1998-1 under, of course, the condition of (4)P.
[5] So can also be c=1.0

4. Determination of principal system.



The reference system is that by which all structure is geometrically described. The principal system is the one that (EAK2000: Σ.3.3.3.[3]) when the model is loaded with the static horizontal seismic forces, in the direction of its axis x or y, would produce no twist on the reference diaphragm.

- The building is loaded in all diaphragms with the seismic forces F_i [6] to the direction of main axes of the reference system.
- The displacements U_{XX}, U_{XY} (Load to axis X: displacement to axis X or Y) and U_{YX}, U_{YY} (Load to axis Y: displacement to axis X or Y) are determined. It is $U_{XY}= U_{YX}$.
- The angle α is calculated for the displacements of point P_0 of the reference diaphragm as $\tan 2\alpha=2U_{XY}/(U_{XX}-U_{YY})$. This angle express the orientation of axes x, y (note the lower case letters) of the principal system. If angle $\alpha<10^0$ then $U_{XX} \sim U_{YY}$ and then it may be taken as α=0.
- The seismic calculation will be performed on the principal system[7], as just defined.

5. Building torsional sensitivity

Torsional sensitivity is defined by the relations (4.1a)[8] and (4.1b) of paragraph 4.2.3.2.(6) EN 1998-1. According to this, in order a building not to be torsional sensitive must, at every level, these two relations to be valid :

$e_{oxi} \le 0.3r_{xi}$ and $e_{oyi} \le 0.3r_{yi}$ $(4.10a),$ $r_{xi} \ge l_{si}$ and $r_{yi} \ge l_{si}$ $(4{,}10b)$ where:

e_{oxi}, e_{oyi} : is the distance between center of inertia and center of mass, calculated on the direction that the relative index specifies, transversally to the other direction.

[6] This expression of EAK2000 on paragraph 3.3.3.[3] and on Σ.3.3.3.[3] indirectly assume that force application point is at the center of mass.
[7] This distinction does not exist on EN 1998-1 paragraph. 4.2.3.2.(6) "*for every analysis direction X and Y*". Nevertheless see EN 1998-1 4.3.3.1.(11)P.
[8] This relation does not exist on EAK 2000.

r_{xi}, r_{yi} : Is the diaphragm (i) radius of torsion of inertia that is defined[9] as $r_{xi} = \sqrt{\frac{c\,U_{yi}}{\theta_{zi}}}$ (respectively for y axis). The U_{yi}, is the y displacement (principal axis) with the model loaded as before, θ_{zi} and c are defined as previously explained in the current text.

l_s : Is the radius of inertia of mass of diaphragm in plan $l_s = \sqrt{\frac{I_{PMi}}{Mi}}$ where I_{PMi} is diaphragm polar moment of inertia on center of mass and M_i the diaphragm mass.

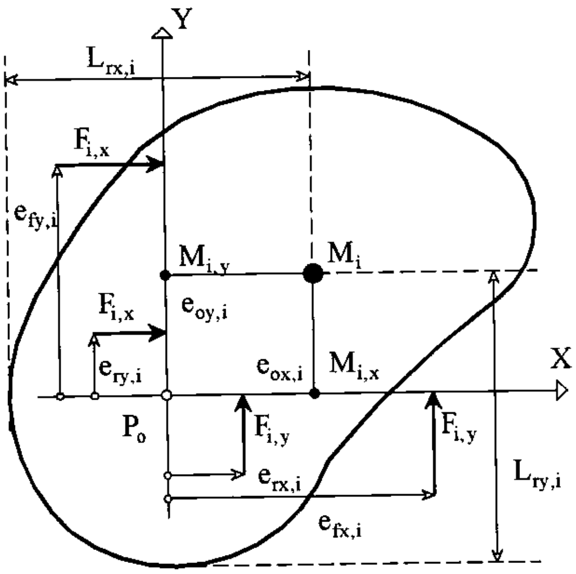
6. Calculation eccentricities.

At every direction are defined 2 eccentricities (either side of centre of mass), to the "elastic" axis, the *max*e_i and *min*e_i:

$\max\ \ e_i = e_{fi} + e_{ai}\ \ and\ \ \ \min\ \ e_i = e_{ri} - e_{ai}$, where

e_{fi}, e_{ri} : Equivalent static eccentricities[10] in order to take account of torsional vibrations of asymmetric buildings for a shifting seismic excitation of the base. In buildings with a symmetry axis their values are zero in the direction of the axis of symmetry. For spectral analysis these eccentricities are ignored. In buildings with no torsional sensitivity (as previously defined) it is allowed these equivalent static eccentricities to be defined approximately as e_{fi}=1.5e_{oi} and e_{ri}=0.5e_{oi}

e_{ai} : Accidental eccentricity[11].



In the general case (under the condition of regularity in elevation - table 4.1 EN 1998-1) the determination shall be according to the (6th) annex ΣΤ of EAK2000 (see drawing beside text), for every diaphragm and every principal direction as follows (note that the index of diaphragm and seismic loading direction is on purposely omitted, for clarity):

- Determine, in the principal system, the ratio:
  - $\varepsilon_0 = \frac{e_0}{l_s}$

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[9] To the N.A. the radius of inertia is calculated in the principal system and not in the reference and equals for x axis $r_{mxi} = \sqrt{r_{xi}^2 + e_{0xi}^2}$ (respectively for y axis). The determination of values follows EAK2000 paragraph 3.3.3.[7] where the radius of inertia is defined in terms of displacements and not stiffness as happens in EN 1998-1 par. 4.2.3.2.(6) (As torsional and bending stiffness are defined differently their equivalency is not certain but at note of par.4.2.3.2.(8)b EN 1998-1 allows the use of alternative definitions).

[10] These eccentricities do not exist in EN1998-1.

[11] Accidental eccentricity is defined in paragraph 4.3.2.(1)P. EN 1998-1. To this paragraph accidental eccentricity is defined as 5% of the width of diaphragm under consideration (i.e. Transversely to the examined direction).

  - $\mu = \frac{r}{l_s}$
  - $l_r = \frac{L_r}{l_s}$

- The eccentricity e_0 shall be taken always with positive sign.
- The L_r is also always positive and is defined, as shown in the previous drawing, by the perimeter of vertical elements (i.e. columns, walls).
- Based on the first uncoupled eigenperiod T of the building, to the direction examined it is defined n as follows: n=1 for $T \leq T_2$ and n=2/3 for $T > T_2$.
- Calculate the acute angle 2ω derived by : $\tan 2\omega = \frac{2\varepsilon_0}{\varepsilon_0^2+\mu^2-1}$ if ω≥0. If ω<0 add, algebraically, the value 90^0.
- Calculate : $A_1 = 1 - \varepsilon_0 \tan\omega$, $A_2 = 1 + \varepsilon_0 \cot\omega$, $\delta_{r1} = \cot\omega - l_r$, $\delta_{r2} = \tan\omega + l_r$
- Then : $l_{12} = \sqrt{\frac{A_2}{A_1}}$, $\varepsilon_{12} = \frac{8\xi^2(1+l_{12})l_{12}^{3/2}}{10^4\left(1-l_{12}^2\right)^2+4\xi^2 l_{12}(1+l_{12})^2}$, where ξ is dumping (in %).
- Then : $R_f = \frac{\sin 2\omega}{2}\left(\frac{1}{A_1^{2n}} + \frac{1}{A_2^{2n}} - 2\varepsilon_{12}\frac{1}{A_1^n A_2^n}\right)^{1/2}$
- Then : $D_r = \frac{\sin 2\omega}{2}\left(\frac{\delta_{r1}^2}{A_1^{2n}} + \frac{\delta_{r2}^2}{A_2^{2n}} + 2\varepsilon_{12}\frac{\delta_{r1}\delta_{r2}}{A_1^n A_2^n}\right)^{1/2}$
- So finally : $e_f = \frac{r^2}{l_s} R_f \geq e_0$, $e_r = \frac{r^2}{l_s} \cdot \frac{1-D_r}{l_r-\varepsilon_0} \leq 0.5e_0$

Note 1: The eccentricity e_r is possible to take negative values in torsionally sensitive systems. The limitations imposed ($e_f \geq e_0$ and $e_r \leq 0.5e_0$) aim to the reduction of unelastic displacements of the softer side and to the plasticity requirements of the stiffer side of the building.

Note 2: Positive values of e_f and e_r measure from P_0 to the directions P_0M_i of projections of the center of mass M_i to the principal axes x or y (see also previous drawing).